Lecture 15

Friday, October 7, 2016 9:15 AM

Increasing/Decreasing Test:

a) If f'(x)>0 on an interval, then f is increasing on that interval.
b) If f'(x)<0 on an interval, then f is decreasing on that interval.

Remark:
We proved this last class using Mean Value Theorem.

Example Find where $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing & decreasing.

Soln $f'(x) = 12x^3 - 12x^2 - 24x$

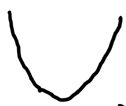
$$= 12x(x^2 - x - 2)$$

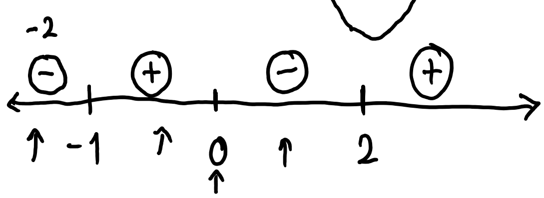
$$= 12x(x-2)(x+1)$$

C.N $f'(x) = 0 \Rightarrow 12x(x-2)(x+1) = 0$

$\Rightarrow 12x = 0,\ x-2 = 0,\ x+1 = 0$

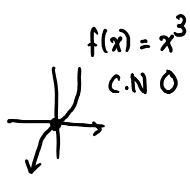
$\Rightarrow x = 0, 2, -1$





Local max @ 0
$f(0) = 5$

Local min @ -1
$f(-1) = 0$



Local min @ 2
$f(2) = -27$

Increasing $(-1, 0) \cup (2, \infty)$

Decreasing $(-\infty, -1) \cup (0, 2)$ //

THE FIRST DERIVATIVE TEST

Spse c is a C.N of a function f.

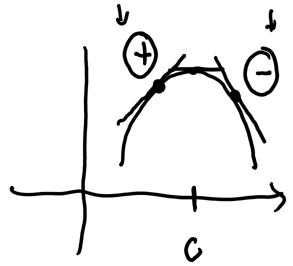
a) If f' changes from ⊕ to ⊖ at c,

then f has a local max at c, (Local max is $f(c)$)

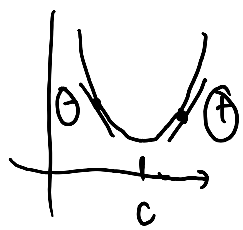
b) If f' changes from $\ominus$ to $\oplus$ at c,
then f has a local min at c.

c) If f' does not change at c (i.e f' is $\ominus$/$\oplus$,
on both sides of c), then f has no
local max or min @ c.

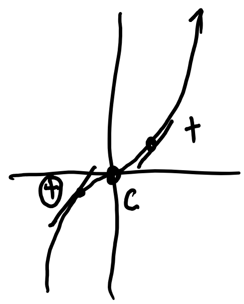
Local max @ c



Local min @ c



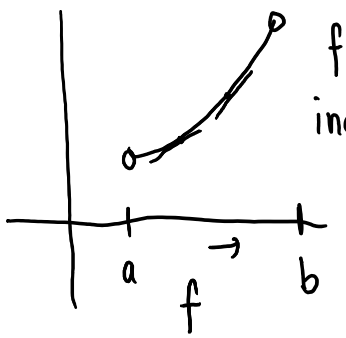
Neither local max or min.



What does f'' tell us about f ?

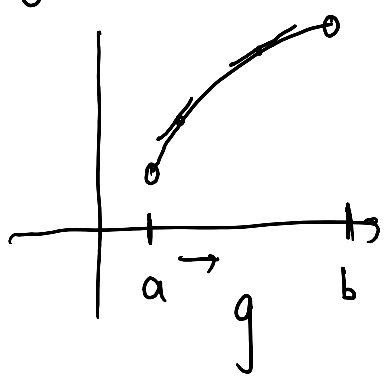
$(f')' = f'' > 0$

f' is increasing



g' is decreasing

$g'' < 0$



Concave Upward

The curve lies above the tangent lines.

Concave Downward

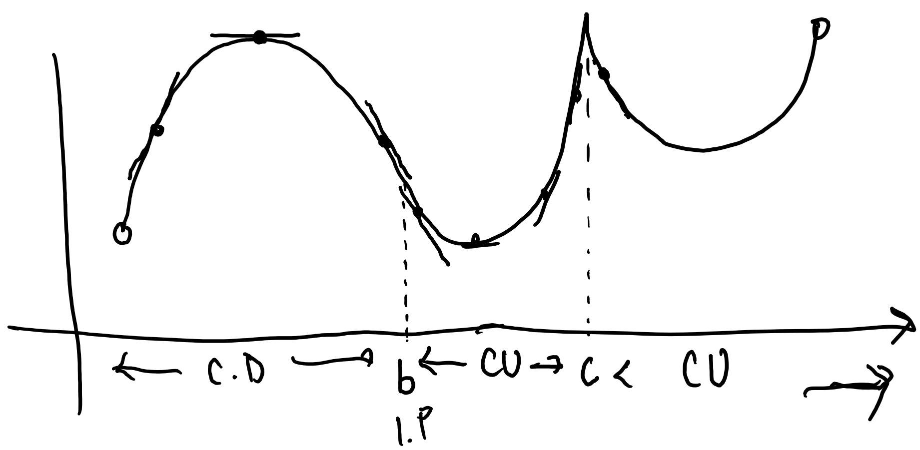
The curve lies below the tangent lines.

Concativity Test

a) If $f''(x) > 0$ on an interval, graph of f is concave upward on that interval

b) If $f''(x) < 0$ " " " , " " " is concave downward on that interval.

<u>DEF</u> A point P on the curve $y = f(x)$ is called an inflection point (I.P) if f is continuous and curve changes from C.U to C.D or from C.D to C.U.



<u>Ex</u> $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

<u>C.N</u> $f'(x) = 0 \Rightarrow 4x^3 - 12x^2 = 0$

$\Rightarrow 4x^2(x-3) = 0 \Rightarrow x = 0, x = 3$

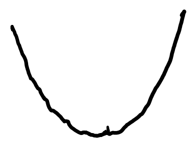
Intervals of incre/decr

$\ominus \quad \ominus \quad \oplus$

$0 \qquad 3$

Inc $(3, \infty)$

Dec $(-\infty, 0) \cup (0, 3)$



Max/min

Neither local max/min @ 0

Local min @ 3 , $f(3) = -27$

Concavity Repeat process w/ second derivatives

$$f''(x) = 12x^2 - 24x = \underbrace{12x}_{+}\underbrace{(x-2)}_{\cancel{+}}$$

$$f''(x) = 0 \Rightarrow 12x^2 - 24x = 0$$

$$\Rightarrow 12x(x-2) = 0 \Rightarrow x = 0, x = 2$$

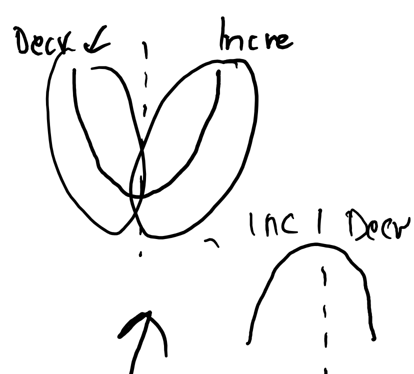
$-1 \quad 1 \quad 3$

$\oplus \quad \ominus \quad \oplus$

$0 \qquad 2$

Con **C.U** $(-\infty, 0) \cup (2, \infty)$

C.D $(0, 2)$

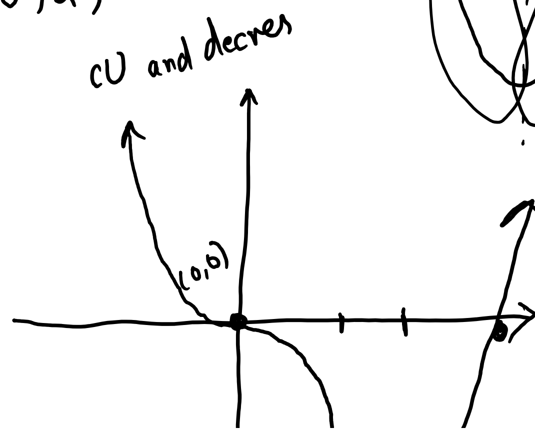


Inflection pt $(0,0), (2,-16)$

$\ominus \; 0 \quad \ominus \; 3 \quad \oplus$

Incr

Decr



− 0 − 3 + Incr

Decr

(3, -27)